DPA on the 'Secure' Permutation in the McEliece PKC

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Outline

Context

Ciphertext permutation

DPA attack

Conclusion
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Conclusion
Communication

Once upon a time ... 

a woman, 

Alice

and a man, 

Bob

who wanted to communicate together.
Attack

But,
they did not want that anyone,

Eve
could understand this message.
That is why, they use cryptology, i.e., the science of secret.

\[ kryptos = \text{secret/hidden} \quad \text{logos} = \text{science} \]
Cryptology

Two concepts:

Cryptography
”Secret writing”

Cryptanalysis
”Analysis of a secret message (cryptogram)”
Symmetric Cryptography

Ceasar
Asymmetric Cryptography
[DH76]
McEliece cryptosystem

[McE78]

- First code-based cryptosystem,
- proposed by Robert McEliece in 1978,
- originally using classical Goppa codes.
Definition (Linear code)
Let $\mathbb{F}_q$ denoted the finite field of $q$ elements. A linear code $\mathcal{C}$ of length $n$ and dimension $k$ is a $k$-dimensional subspace of $\mathbb{F}_q^n$.

Definition (Generator matrix)
Let $\mathcal{C}$ be a $[n, k]_q$-linear code. Let $\mathcal{G} \in \mathcal{M}_{k,n}(\mathbb{F}_q)$. We call $\mathcal{G}$ a generator matrix of $\mathcal{C}$ iff $\mathcal{G}$-rows are basis vectors of $\mathcal{C}$. 
McEliece key generation

[McE78]

**Inputs:** $n$ and $t$ two integers.

1. Choose a linear code $\mathcal{C}$ of length $n$ and $t$-correcting. 
   $k$: dimension of $\mathcal{C}$.
2. Take one generator matrix $G \in M_{k,n}(\mathbb{F}_2)$ of $\mathcal{C}$.
3. Randomly choose one invertible matrix $S \in M_{k,k}(\mathbb{F}_2)$.
4. Randomly choose one permutation matrix $P \in M_{n,n}(\mathbb{F}_2)$.
5. Compute the generator matrix given by $\tilde{G} = S \cdot G \cdot P$.
6. $s_k \leftarrow (S, G, P, \mathcal{C})$
7. $p_k \leftarrow (\tilde{G}, t)$
8. Return $(p_k, s_k)$.

**Outputs:** Public key $p_k = (\tilde{G}, t)$ and private key $s_k = (S, G, P, \mathcal{C})$. 
McEliece encryption

[McE78]

**Inputs:** Public key $p_k = (\tilde{G}, t)$, message $M \in \mathbb{F}_2^k$.

1. Encode the message $C = M \cdot \tilde{G}$.
2. Randomly choose an error vector $E \in \mathbb{F}_2^n$ of weight $w_H(E) = t$.
3. Compute $\tilde{C} = C \oplus E$.
4. Return $\tilde{C}$.

**Output:** Ciphertext $\tilde{C} \in \mathbb{F}_2^n$ associated to $M$. 
McEliece decryption

[McE78]

**Inputs:** Private key $s_k = (S, G, P, \Gamma)$, ciphertext $\tilde{C} \in \mathbb{F}_2^n$.

1. Compute $\tilde{C}_p = \tilde{C} \cdot P^{-1}$.
   
   i.e. $\tilde{C}_p = M \cdot S \cdot G \oplus E \cdot P^{-1}$

2. Decode $\tilde{C}_p$ to obtain $M \cdot S \cdot G$.

3. Get $\tilde{M} = M \cdot S$ from $M \cdot S \cdot G$.

4. Compute $M = \tilde{M} \cdot S^{-1}$.

5. Return $M$.

**Output:** Plaintext $M \in \mathbb{F}_2^k$ associated to $\tilde{C}$. 
Side-Channel Attack (SCA)

Definition (SCA)
Exploit the laws of physics phenomenons to obtain some information contained in channels associated to an implementation (software or hardware).

Definition (DPA)
Use several power traces for a same secret/private key, compute the average to avoid noise (very often), and find a pattern on power traces depending on the secret/private key in order to recover it.
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'Simple' permutation

Example

\[ \tilde{C}_p \]

\[ \tilde{C} \]

\[ \tilde{C} \]

\[ \tilde{C}_p \]

\[ \tilde{C} \]

\[ \tilde{C} \]

\[ \tilde{C} \]
'Simple' permutation

Algorithm

**Inputs:** Private permutation matrix $\mathcal{P}^{-1} \in M_{n,n}(\mathbb{F}_2)$ represented by a lookup table $t^{\mathcal{P}^{-1}}$, ciphertext $\tilde{C} \in \mathbb{F}_2^n$.

For $i = 0$ to $n - 1$

\begin{align*}
  j &= t^{\mathcal{P}^{-1}}_i \\
  \tilde{C}_{p_i} &= \tilde{C}_j
\end{align*}

Endfor

Return $\tilde{C}_p$.

**Output:** Permuted ciphertext $\tilde{C}_p \in \mathbb{F}_2^n$. 
'Secure' permutation [STMOS08]

Algorithm

Inputs: Private permutation matrix $\mathcal{P}^{-1} \in \mathcal{M}_{n,n}(\mathbb{F}_2)$ represented by a lookup table $t^{\mathcal{P}^{-1}}$, ciphertext $\tilde{C} \in \mathbb{F}_2^n$.

1. For $i = 0$ to $n - 1$
2. $j = t^{\mathcal{P}^{-1}}_i$
3. $\tilde{C}_{p_i} = 0$
4. For $h = 0$ to $n - 1$
5. $k = \tilde{C}_{p_i}$
6. $\mu = \tilde{C}_h$
7. $s = j \oplus h$
8. $s \mid= s \gg 1$
9. $s \mid= s \gg 2$
10. $s \mid= s \gg 4$
11. $s \mid= s \gg 8$
12. $s \mid= s \gg 16$
13. $s \& = 1$
14. $s = \sim (s - 1)$
15. $\tilde{C}_{p_i} = (s \& k) \mid ((\sim s) \& \mu)$
16. Endfor
17. Endfor
18. Return $\tilde{C}_p$

Output: Permuted ciphertext $\tilde{C}_p \in \mathbb{F}_2^n$. 
'Secure' permutation [STMOS08]

Examples

<table>
<thead>
<tr>
<th>Steps</th>
<th>Test hypotheses</th>
</tr>
</thead>
<tbody>
<tr>
<td>7: $s = j \oplus h$</td>
<td>$100\ldots0$</td>
</tr>
<tr>
<td>8: $s \mid= s \gg 1$</td>
<td>$1100\ldots0$</td>
</tr>
<tr>
<td>9: $s \mid= s \gg 2$</td>
<td>$111100\ldots0$</td>
</tr>
<tr>
<td>10: $s \mid= s \gg 4$</td>
<td>$11\ldots100\ldots0$</td>
</tr>
<tr>
<td>11: $s \mid= s \gg 8$</td>
<td>$11\ldots100\ldots0$</td>
</tr>
<tr>
<td>12: $s \mid= s \gg 16$</td>
<td>$11\ldots1$</td>
</tr>
<tr>
<td>13: $s &amp; = 1$</td>
<td>$00\ldots01$</td>
</tr>
<tr>
<td>14: $s = \sim (s - 1)$</td>
<td>$11\ldots1$</td>
</tr>
</tbody>
</table>
Weakness [PRDCF16]

Leakage Step 15:

\[ \tilde{C}_{p_i} = \begin{cases} (s \& k) & \text{true only if } s=11...1 \\ \text{else false} \\ ((\sim s) \& \mu) & \text{true only if } s=00...0 \\ \text{else false} \end{cases} \]

Giving:

\[ k = 0 \quad \text{and} \quad k = \tilde{C}_j \]
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Attack bench [PRDCF16]
Traces analysis [PRDCF16]

- Apply a Hamming weight of individual bits leakage model: $H_i \in \{0, 1\}$,
- Use correlation coefficient to test our hypotheses compared with measurements,
- Good hypothesis if the coefficient is (almost) 1 or -1,
- Average of 500 traces per ciphertext hypothesis to avoid noise,
- Chosen ciphertexts as every vectors of weight 1.
Traces examples [PRDCF16]
Countermeasure [PRDCF16]

**Algorithm**

**Inputs:** Private permutation matrix $\mathcal{P}^{-1} \in \mathcal{M}_{n,n}(\mathbb{F}_2)$ represented by a lookup table $t^{\mathcal{P}^{-1}}$, ciphertext $\tilde{\mathcal{C}} \in \mathbb{F}_2^n$ and private generator matrix $\mathcal{G}$ of $\Gamma(\mathcal{L}, \mathcal{G})$.

1. Randomly choose $B \in \Gamma(\mathcal{L}, \mathcal{G})$
2. $B_p = B \cdot \mathcal{P}$
3. $\tilde{\mathcal{C}}' = \tilde{\mathcal{C}} \oplus B_p$
4. **For** $i = 0$ **to** $n - 1$
5. $j = t^i_{\mathcal{P}^{-1}}$
6. $\tilde{\mathcal{C}}_{p_i}' = 0$
7. **For** $h = 0$ **to** $n - 1$
8. $k = \tilde{\mathcal{C}}_{p_i}'$
9. $\mu = \tilde{\mathcal{C}}_h'$
10. $s = j \oplus h$
11. $s \mid = s \gg 1$
12. $s \mid = s \gg 2$
13. $s \mid = s \gg 4$
14. $s \mid = s \gg 8$
15. $s \mid = s \gg 16$
16. $s \& = 1$
17. $s = \sim (s - 1)$
18. $\tilde{\mathcal{C}}_{p_i}' = (s \& k) | ((\sim s) \& \mu)$
19. **Endfor**
20. **Endfor**
21. **Return** $\tilde{\mathcal{C}}_p'$

**Output:** Permuted ciphertext $\tilde{\mathcal{C}}_p' \in \mathbb{F}_2^n$ masked by a codeword.
Countermeasure [PRDCF16]

Main idea

From masked ciphertext to masked permuted ciphertext:

\[ \tilde{C}_p' = \tilde{C}' \cdot P^{-1} \]
\[ = (\tilde{C} \oplus B_p) \cdot P^{-1} \]
\[ = \tilde{C} \cdot P^{-1} \oplus (B \cdot P) \cdot P^{-1} \]
\[ = \tilde{C}_p \oplus B. \]

From masked permuted ciphertext to the same syndrome than non-masked ciphertext:

\[ S = \tilde{C}_p' \cdot H^T \]
\[ = (\tilde{C}_p \oplus B) \cdot H^T \]
\[ = \tilde{C}_p \cdot H^T \oplus \underbrace{B \cdot H^T}_{=0} \]
\[ = \tilde{C}_p \cdot H^T. \]
Countermeasure [PRDCF16]

Trace example
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Conclusion

- DPA against a ‘secure’ permutation algorithm (countermeasure for cache-memory attack),
- Simple masking countermeasure (with $n$ more bits and not a huge amount of additional computations),
- DPA not depending on the code structure so possible for others linear codes than Goppa codes.
DPA on the ’Secure’ Permutation in the McEliece PKC

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Thank you for your attention!
References


Pearson’s correlation coefficient

We used for correlation analyses:

\[
r_{H,X}(\eta) = \frac{\sum_{i=1}^{N} [(X_i(\eta) - \bar{X}(\eta))(H_i - \bar{H})]}{\sqrt{\sum_{i=1}^{N} [X_i(\eta) - \bar{X}(\eta)]^2 \sum_{i=1}^{N} (H_i - \bar{H})^2}}
\]

where \( r_{H,X}(\eta) \) is the Pearson’s correlation coefficient for \( \eta \)-th sample (measured during execution of the cryptographic algorithm), \( N \) is a number of measured traces, \( X_i(\eta) \) is a value of \( \eta \)-th sample measured during \( i \)-th measurement (\( i \)-th trace), \( \bar{X}(\eta) \) is a mean value of corresponding \( \eta \)-th samples (from all traces), \( H_i \) is a hypothesis of power consumption for one bit of input data corresponding with \( i \)-th measurement (\( i \)-th trace) and \( \bar{H} \) is a mean value of all hypotheses \( H_i \).